Orbit Determination for Asteroid 2003 LS3

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***Abstract*—Asteroid 2003 LS3, which is of interest because of its orbit which comes close to the Earth, was observed and analyzed over the course of a month in order to determine an accurate model of its future orbit. The Right Ascension and Declination coordinates, as well as the apparent magnitude, of the asteroid were recorded in 25 separate observations, and these data were employed to determine 2003 LS3's classical elliptical orbital elements, as well as its predicted size. The asteroid's path was also integrated long-term, based on our model, in order to check for possible future collisions with the Earth.**

I. Introduction

NASA has discovered more 14,000 objects in the solar system whose orbits come close to the Earth[1]. Though there are many reasons for observing and characterizing these Near-Earth Objects (NEOs), the primary motivation for this project was to analyze the potential hazard that 2003 LS3 may one day pose to terrestrial life—in the past, asteroids colliding with Earth have significantly impacted the biosphere. Because the solar system contains several massive bodies, it is probable that asteroids will be perturbed from its expected orbit. It is therefore necessary to constantly monitor these objects.

Another reason to study NEOs is for scientific research. Current observations show no hazardous asteroid on track to collide with Earth; however, in the event that one is discovered, NASA seeks to be prepared with comprehensive information about the characteristics of such asteroids. Plans have been developed to capture one of these NEOs and bring it into orbit around the Earth for close study. In addition, private companies are developing technologies for the purpose of mining resources from asteroids for profit, and eventual human exploration and colonization. Their work hinges on the accuracy of these asteroid profiles and orbit models.

2003 LS3 was imaged using three telescopes in the United States and Chile: a 16” telescope located at the Leitner Observatory on the Yale University campus in New Haven, Connecticut; the Prompt6 SkyNet telescope at the Cerro Tololo Inter-American Observatory in Chile; and a 0.61 m MPC U69 telescope in Auberry, California. Celestial coordinates of the asteroid were analyzed using the Method of Gauss to identify preliminary vector elements of the orbit. This orbit model was then optimized to fit the observations using a genetic algorithm. The potential for error in our model must be noted due to the limited set of observations across one month. The process of long term integration of the orbit accounted for the gravitational attraction of the Sun, Jupiter, and Saturn.

II. Methods

A. Telescopes

The following are the telescopes used for data collection along with their specifications.

TABLE I

Telescope	Field of View	Lat, Long	Observatory Code
Leitner 16in	24’ x 24’	$42.31°, -72.93°$	797
Skynet Prompt6	15’ x 15’	$-30.17°, -70.81°$	807
iTelescope T24	31.8’ x 31.8’	$37.07°, -119.4°$	U69

B. Observation Procedures

The telescope was connected to a computer using TheSky X, which was used to control the telescopes pointing and take its images. After the telescope and camera were connected to the computer and the temperature of the CCD camera was set at -5 °C, there were two main calibration tasks to do. First, the telescope had to be synchronized to the sky by pointing calibration. To do this, we pointed the telescope at a bright star (usually Altair) that was close to the predicted coordinates of the asteroid for that night. Then we took test images of the bright star, and adjusted the telescope's aim until the star was in the center of the image. Then we synchronized TheSky X's map of the sky to the bright star's location.

After doing pointing calibration, we aimed the telescope at the coordinates for the asteroid predicted by NASA JPL's ephemeris. By pointing at the patch of sky containing the asteroid, we could focus the telescope by adjusting its secondary mirror until the stars in the test images looked like small, sharp points. With the telescope calibrated and focused, we began taking series of 60-second exposures centered on the JPL ephemeris for 2003 LS3 at the observing time. Exposures were taken with a luminance and a red filter. Usually, we took around 20 luminance images followed by around 9 red images, then ended the observing session with another 15 luminance images.

C. Data Analysis

1) Least Squares Plate Reduction and Centroid Determination: Following the collection of usable .fit image sets, the asteroid coordinates were analyzed in the images primarily using Astrometry.net for LSPR field solving the images, and the SAOImage DS9 astronomical application for recording the centroids and Right Ascension/Declination coordinates. The first online-based application identifies formations of stars in order to accurately match RA/Dec values to the images dimensions, and returns .fits images with embedded coordinates to the user. These solved images are then analyzed with DS9, returning the celestial coordinates of a selected centroid region of the image.

2) Method of Gauss: After identifying the right ascension and declination of each centroid in our data set, the preliminary vector orbital elements needed to be determined. Several methods exist which solve this problem with varying accuracy. Here, the Method of Gauss was used to solve for the orbit.[3] The Method of Gauss utilizes 3 observations, each on different days—$\{\alpha_i, \delta_i, t_i\}$ for $i = 1, 2, 3$. It also utilizes Newton's law of universal gravitation:

$$\ddot{\mathbf{r}} = \frac{-\mu\mathbf{r}}{r^3}$$

which shows that an object with position $\mathbf{r}$, velocity $\dot{\mathbf{r}}$ will move in an elliptical orbit with the sun at one of its foci. This acceleration function and the 3 data points, along with Gauss's 13-step iterative method outputted the vectors $\mathbf{r}$ and $\dot{\mathbf{r}}$ for the value t_2.

3) Orbital Elements: Because right ascension and declination output equatorial vector orbital elements, they needed to be rotated into ecliptic coordinates in order to determine the classical orbital elements.To accomplish this, the unit ecliptic vector $\hat{\mathbf{r}}_{equ}$ Must be multiplied by the rotation matrix $\mathbf{M}$:

$$\hat{\mathbf{r}}_{ecl} = \mathbf{M} \cdot \hat{\mathbf{r}}_{equ}$$

The rotation matrix uses an angle of ϵ, the angle between the equator and the ecliptic on the summer solstice, to rotate the vector.

The orbital elements were created in a computer program. The program took in the asteroid's position and velocity vector as inputs and then calculated the classical orbital elements from those. The variables for these elements are as follows:

TABLE II
CLASSICAL ORBITAL ELEMENTS

Element	Definition
a	Semimajor axis of the orbit
e	Eccentricity
i	Inclination with respect to the xy plane
Ω	Longitude of the ascending node
ω	Argument of the perihelion
T	Specified Julian Date: 2457597.649

The elements are calculated with the following, where $\mathbf{r}$ and $\dot{\mathbf{r}}$ are position and velocity respectively at a given time, $\hat{z}$ is the unit vector along the positive z-axis, $\hat{x}$ is the unit vector along the x-axis, and N is a vector pointing to the ascending node.

$$\mathbf{h} = \mathbf{r} \times \dot{\mathbf{r}}$$

$$\mathbf{e} = \dot{\mathbf{r}} \times \mathbf{h} - \frac{\mathbf{r}}{r}$$

$$a = \frac{1}{\frac{2}{r} - \dot{r}^2}$$

$$\mathbf{N} = \hat{z} \times \mathbf{h}$$

$$\Omega = \arccos \frac{\mathbf{N} \cdot \hat{x}}{N}$$

$$\omega = \arccos \frac{\mathbf{N} \cdot \mathbf{e}}{eN}$$

When finding Ω and ω, it is important to multiply by -1 and add 360° if $\mathbf{N}_y < 0$ (for Ω) or if $\mathbf{e}_z < 0$ (for ω). This is because the dot product finds the smallest angle between two vectors, but the orbital elements require the angle to be in standard form (clockwise only).

4) Orbit Optimization with Genetic Algorithm: We optimized the orbit of 2003 LS3 using a genetic algorithm, in which the computer randomly makes a population of asteroids with slightly different positions and velocities from the preliminary model. Then the computer determines the most fit model and destroys all the rest, and then repeats the whole process a fixed number of times.

To start, we gathered all the astrometric data we had taken on 2003 LS3 into a table of Julian Dates and matching Right Ascension and Declination measurements (see Table VIII). The optimization program read these values into a table. After taking in the initial position and velocity at a particular observation in the data set, the genetic algorithm created 100 more asteroids with positions randomly generated such that every component of the position fit a normal distribution around the preliminary position vectors component. Then the program used the Fourth Order Runge-Kutta method (RK4) of integration to generate a prediction of Right Ascension and Declination at every time during which we observed. The program then compared these values with the observed values, and took the root mean square (σ_{RMS}) of the data[1]. Once every asteroid had calculated a root mean square, the program deleted all but the one asteroid with the lowest root mean square. Then the program created 100 more asteroids, this time with the same position as the remaining asteroid but with a velocity determined randomly just as before. After calculating the root mean square for all asteroids and removing all but the asteroid with the best fit to the data, the program repeated this process over again. After each generation, the standard deviation of the random normal function (which was used to generate positions and velocities) was multiplied by 0.9 to allow for more fine adjustment of the vectors.

After about 10 generations, we took the position and velocity vectors that best fit our data and repeated the program but with a significantly lower standard deviation (approximately 0.01 times the original standard deviation). This caused the algorithm to find more accurate solutions with fewer generations.

[1]The formula for root mean square is $\sigma_{RMS} = \sqrt{\frac{\sum(x_i - \hat{x_i})}{N-3}}$, where N is the number of observations, x_i is every observation, and $\hat{x}_i$ is the predicted value for each observation.

5) Long-Term Integration: We made use of a Python package called Rebound when creating a long-term integrator for 2003 LS3. This integrator is designed to be more accurate than RK4 as well as more computation-efficient[2]. First, we created a simulation object and added the Sun, Earth, Jupiter, and Saturn into its list of particles. Then we added the asteroid, with the best-fit position and velocity vector that was output by the orbit optimization program. Next, we put the program through a loop during which it integrated until a collision occurred or until a million years were simulated. During this simulation, the distance of the asteroid from the Earth was recorded in a text file whenever it had a new closest approach.

After this first integration was done, we generated new best-fit vectors for the asteroid by randomly altering our data points by a random normal function with standard deviation 1.4 arcseconds (the smallest resolution available on the 16" telescope from Leitner). This created a set of vectors with slight deviation from our current best model based on the error of the telescope. Then we ran the integrator with each of these new vectors to find out how many of them resulted in a collision with the Earth.

6) Photometry: Photometry was performed with another combination of applications, in this case SAOImage DS9 and MaximDL. The DS9 contains catalogs of star information, which we referred to in order to assign reference magnitudes of nearby stars in MaximDL.

Figure 1: Sample MaximDL Photometry Analysis

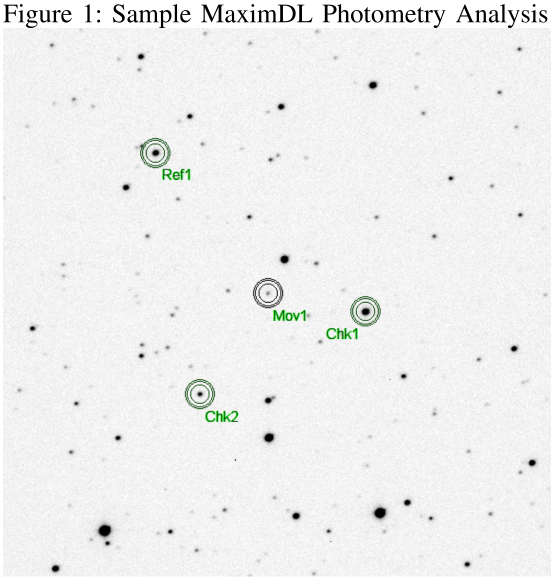

TABLE III
SAMPLE MAGNITUDE MEASUREMENTS

Julian Date	Ref1 Mag	Chk2 Mag	Chk3 Mag	Asteroid Mag
2457587.835	14.418	16.052	15.601	16.769
2457587.841	14.418	16.037	15.533	16.8
2457587.846	14.418	16.003	15.489	16.797

[2]The package also can get the locations and velocities of the celestial bodies in the solar system from JPL's ephemeris.

MaximDL then generated the relative apparent magnitude of the selected asteroid for each image. Knowing these, it was then possible to determine a predicted absolute magnitude and size range of the asteroid, as follows.

The relation between absolute and apparent magnitude is described as $M_{app} - \Delta M = M_{abs}$, where:

$$\Delta M = -2.5log((\frac{b_1}{b_2})^2)$$

In this situation, b_1 and b_2 represent the brightnesses of a reference star, but these brightness values are inversely related to the squares of the distances to the asteroid from the Earth and the Sun respectively, so the equation would then appear as:

$$\Delta M = -2.5log((\frac{1/\vec{\rho}}{1/\vec{r}})^2)$$

These distance values for a particular Julian Date were easily generated after optimization of the orbit model, and after ΔM was found it was added to the average apparent magnitude of a series of Julian Dates about the one used for distance reference, Such as the one in Table II, in order to receive an estimate of the absolute magnitude of the asteroid. The size of the asteroid was then estimated using JPL Horizons Absolute Magnitude Glossary for corresponding magnitudes and sizes. [2]

D. Uncertainty

When taking exposures of the asteroid, between 30 and 90 seconds are required to collect enough light for it to be visible. However, the asteroid moves in relation to the celestial sphere during this time. If the asteroid is moving quickly in the sky, the asteroid can cause streaks in the image, which would cause more error when finding the centroid. Our asteroid did not move quite this fast in the sky, but it was fast enough that it was difficult to median combine the images without causing streaks. Because of this, we could not median combine the images for the data from Skynet Prompt6. When median combining is not possible, we get less signal compared to the ambient noise and the centroid is less accurate.

There is error once the pictures are taken because we relied on the LSPR method from Astrometry.net to solve the images for Right Ascension and Declination. The sky is not perfectly flat so any attempt to overlay a grid on the rectangular image will cause some amount of warping. Even the RK4 integration method can compound error over time when integrating, which also can lead to uncertainty in measurements.

III. RESULTS

A. Coordinate and Photometry Measurements

TABLE IV
COORDINATE AND PHOTOMETRY MEASUREMENTS

Julian Date	RA(hrs)	Dec(deg)	Mag
2457587.835	20 27 32.71	+05 24 34.27	16.77
2457587.841	20 27 33.14	+05 24 32.83	16.80
2457587.846	20 27 33.45	+05 24 32.15	16.80
2457590.833	20 31 16.23	+05 11 32.84	16.32
2457590.840	20 31 16.83	+05 11 29.50	16.96
2457590.847	20 31 17.33	+05 11 27.75	16.76
2457590.852	20 31 17.66	+05 11 25.66	15.99
2457590.860	20 31 18.31	+05 11 22.41	16.05
2457597.616	20 40 34.00	+04 10 59.99	16.15
2457597.623	20 40 34.97	+04 10 55.99	16.16
2457597.629	20 40 34.97	+04 10 50.99	16.70
2457597.649	20 40 36.98	+04 10 36.98	16.38
2457597.655	20 40 36.98	+04 10 32.99	15.94
2457597.661	20 40 38.00	+04 10 27.98	15.85
2457600.765	20 45 17.92	+03 28 36.89	16.18
2457600.771	20 45 18.37	+03 28 32.27	16.13
2457600.778	20 45 18.96	+03 28 25.13	16.46
2457600.784	20 45 19.49	+03 28 19.85	16.10
2457600.800	20 45 20.89	+03 28 04.74	16.36
2457600.804	20 45 21.21	+03 28 00.93	16.24
2457604.682	20 51 42.26	+02 20 23.20	16.04
2457604.688	20 51 42.75	+02 20 16.58	16.39
2457604.691	20 51 42.99	+02 20 13.40	15.95
2457604.695	20 51 43.39	+02 20 08.06	15.79
2457604.699	20 51 43.67	+02 20 04.99	15.93

B. Asteroid Magnitude and Size

The absolute magnitude of the asteroid was estimated at around 18.82. The corresponding size range, according to the JPL Magnitude Glossary, to an asteroid of magnitude 19 would be a diameter of around 420 m – 940 m.

C. Orbit Model

Our Gauss method and finalized orbit model yielded the following initial orbit conditions for Julian Date 2457597.649:

$$\mathbf{r} = \langle 0.80032, -1.0527, 0.12333 \rangle$$

$$\dot{\mathbf{r}} = \langle 0.74294, 0.77740, -0.18153 \rangle$$

The position ($\mathbf{r}$) vector is measured in AU, and the velocity vector ($\dot{r}$) in AU per year. The initial RMS value for these measurements had a value of 0.0108.

The classical orbital elements, based on these determined position and velocity vectors, resulted as follows:

We also found a graphical representation of the orbit (See Figure 2).

TABLE V
CLASSICAL ORBITAL ELEMENTS

Element	Value
a	$3.1581 \pm .001$
e	$0.5958 \pm .001$
i	$10.305° \pm .008$
Ω	$158.10° \pm .029$
ω	$178.09° \pm .109$
T	2457597.649

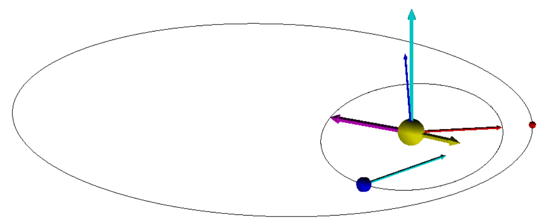

Figure 2. A rendering of the orbit model for 2003 LS3.

D. Long-Term Orbit

The long-term integrator for 2003 LS3 using our best-fit position and velocity vectors predicted the asteroid colliding with Earth. Upon further investigation, with two other initial orbits generated randomly as described in the ”Long Term Orbit” section of our Method, we got the following results:

TABLE VI
LONG-TERM INTEGRATION RESULTS

Trial Number	Last Calculated Distance (AU)	JD	Calendar Year
1	$2.8695 \times 10{-}5$	3915282	CE 6007
2	2.0674×10^{-5}	4049737	CE 6375
3	2.7983×10^{-5}	3827928	CE 5768

The integration is calculation-heavy, so it takes over an hour to run each of these integrations. This is why the sample size is so small, and therefore why it is impossible to state the probability of collision with the Earth within the next million years. However, assuming our data is somewhat accurate, the fact that even slightly altered models still collided with Earth within 300 years of the original prediction implies a collision is a possibility.

E. Comparison with Accepted Parameters

In comparing our model predictions to the existing JPL Horizons 2003 LS3 model information, we found that we did not have any significantly contradictory deviations from the previous parameters.

As can be seen in the above Table VII, the classical orbital elements of semimajor axis and eccentricity are more skewed in comparison, though this is likely due to differing coordinate measurements, as can be seen below.

Table VIII displays disagreements between our observed values and the predicted JPL values. They are not too significantly off, though some clear loss of precision in our

TABLE VII
ORBITAL ELEMENTS COMPARISON

Element	Modeled Estimate	JPL Estimate
a	$3.1581 \pm .001$	2.65446
e	$0.5958 \pm .001$	.52374
i	$10.305° \pm .008$	9.525°
Ω	$158.10° \pm .029$	158.11°
ω	$178.09° \pm .109$	175.184°

TABLE VIII
COORDINATE AND PHOTOMETRY MEASUREMENTS

Julian Date	Observed RA	Observed Dec	JPL RA	JPL Dec
2457597.616	20 40 34.00	+04 10 59.99	20 40 34.33	+04 11 01.1
2457597.623	20 40 34.97	+04 10 55.99	20 40 34.89	+04 10 56.1
2457597.629	20 40 34.97	+04 10 50.99	20 40 35.37	+04 10 51.7
2457597.649	20 40 36.98	+04 10 36.98	20 40 36.96	+04 10 37.3
2457597.655	20 40 36.98	+04 10 32.99	20 40 37.43	+04 10 33.0
2457597.661	20 40 38.00	+04 10 27.98	20 40 37.90	+04 10 28.6

observations is causing most of the discrepancy. However, after running our genetic algorithm in order to generate as accurate an orbit as possible, slight errors in observations are not likely to make too large a difference, as demonstrated by our minimal standard deviations in orbital elements. Thus, we stay confident in our own generated orbit model, and submit the possibility that there is a significant enough difference between our model and the JPL model that the JPL Horizons information is not quite completely accurate.

IV. CONCLUSION

Over the course of four weeks, asteroid 2003 LS3 was observed on five different nights. For each night, least square plate reduction was used to convert the pixel values to right ascension and declination coordinates. These values, in addition to the JPL ephemeris range vector from Earth to the sun, allowed the Method of Gauss to be utilize to determine preliminary vector orbital elements. After running a genetic optimization algorithm, we found a model for the asteroid's orbit that fits our data with a σ_{RMS} of 0.010801. Utilizing photometry procedures, we also approximated the absolute magnitude of the asteroid at a value of around 18.82, with a size of about 420m – 940m.

Through long-term integration, it was determined that 2003 LS3 will collide with the earth within 5,000 years. With the σ_{RMS} of our orbit model, our observation have an uncertainty of ± 39 arcseconds for standard deviation from our observed coordinates. This uncertainty, however, likely does not significantly impact the accuracy of our determined model, as the genetic algorithm run on the initial orbit conditions achieved orbital elements very similar to those that had previously been modeled. Thus, our model raises concern about this future collision, and warrants further investigation and data collection for verification.

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